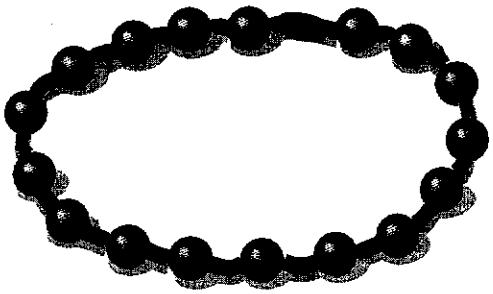
Fractions

The word ***fraction*** means "part of a whole." The word comes from the Latin word ***fractio***, meaning "to break into pieces." In math, a fraction means one or more parts of a whole or a set (see p. 12).



This necklace is made up of beads. Each bead is a member of the "set," or necklace. Since 17 beads make up this necklace, each bead is 1 part of 17 total necklace parts, or $\frac{1}{17}$.

A fraction has two parts, a ***denominator*** and a ***numerator***. The denominator is the numeral written under the bar and tells the number of parts a whole is divided into. The numerator is the numeral written above the bar. The numerator tells the number of parts of the whole that are being counted.

numerator	number of parts counted	1
denominator	total parts of the whole or set	17

A fraction is another way of writing a division problem. The fraction $\frac{1}{4}$ means $1 \div 4$. The denominator of a fraction can never be 0 because you cannot divide numbers by 0 (see p. 37).

Proper Fractions

When the numerator of a fraction is less than the denominator, the fraction is called a ***proper fraction***.

$\frac{1}{2}$ $\frac{2}{5}$ $\frac{3}{8}$ $\frac{4}{9}$ $\frac{5}{11}$

The value of a proper fraction is always less than one.

Improper Fractions

When the numerator of a fraction is greater than or equal to the denominator, the fraction is called an ***improper fraction***.

$\frac{3}{2}$ $\frac{4}{3}$ $\frac{5}{4}$ $\frac{6}{5}$ $\frac{7}{6}$ $\frac{8}{8}$

The value of an improper fraction is always greater than or equal to one.

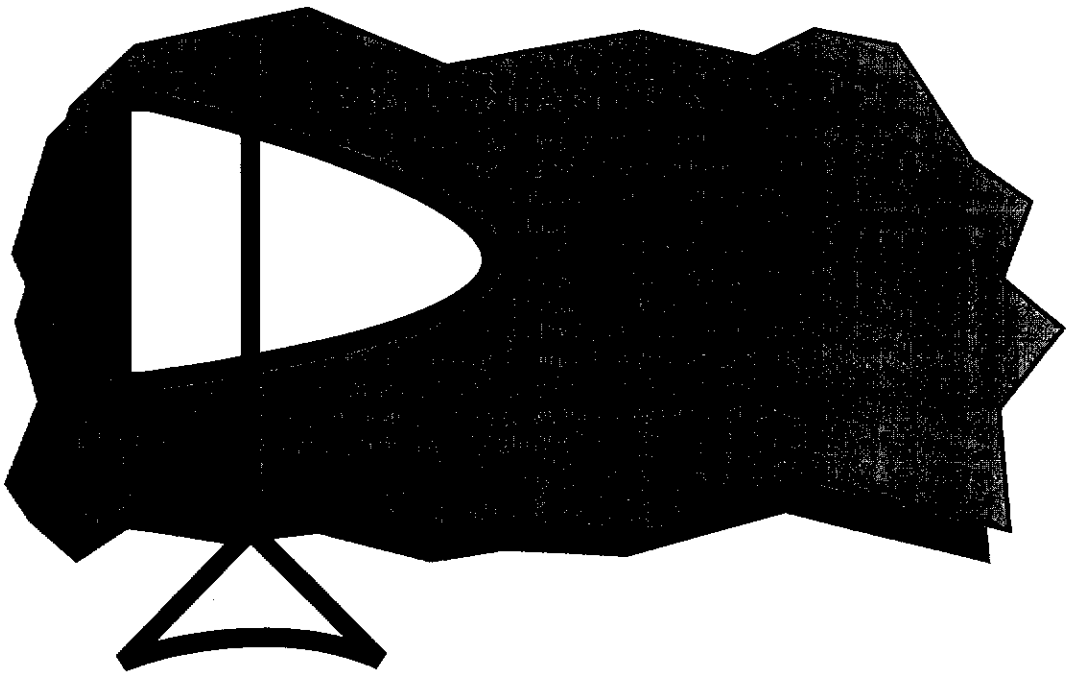
Mixed Numerals

Mixed numerals combine whole numbers and fractions. The values of mixed numerals can also be written as ***improper fractions***. To write a mixed numeral as an improper fraction, multiply the whole number by the denominator of the fraction, then add the numerator. Use your answer as the new numerator and keep the original denominator.

$$1\frac{1}{2} = \frac{(2 \times 1) + 1}{2} = \frac{3}{2} \qquad 2\frac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{11}{4}$$

To change an improper fraction to a mixed numeral, divide the numerator by the denominator. Then place the remainder over the old denominator.

$$\frac{3}{2} = 2\overline{)3}\,\begin{matrix}1\\ -2\\ \hline 1\end{matrix} = 1\frac{1}{2} \qquad \frac{11}{4} = 4\overline{)11}\,\begin{matrix}2\\ -8\\ \hline 3\end{matrix} = 2\frac{3}{4}$$



Common Denominators

Many fractions have ***common denominators***. That means that the numbers in their denominators are the same.

$\frac{1}{2}$ $\frac{3}{2}$ $\frac{5}{2}$

To find common denominators, ① find the ***least common multiple*** (see p. 23) for the denominators of the fractions you are comparing. Compare:

$\frac{1}{2}$ and $\frac{2}{3}$ Answer: least common multiple is 6

② Divide the common multiple by the denominators.

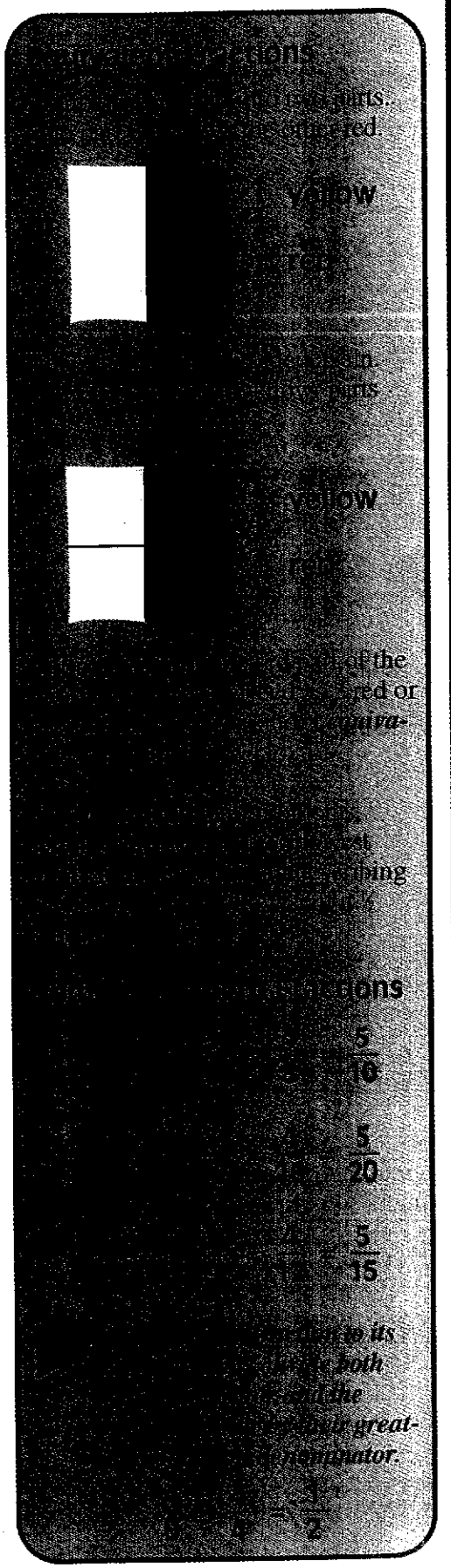
$2\overline{)6}$ $\quad 3$ $\qquad 3\overline{)6}$ $\quad 2$

③ Multiply the quotients by the old numerators to calculate the new numerators.

$3 \times 1 = 3$ $\qquad 2 \times 2 = 4$

④ Place the new numerators over the common denominator.

$\frac{3}{6}$ $\qquad \frac{4}{6}$



Adding and Subtracting Fractions

To add fractions, the fractions must have ***common denominators***. To add fractions with common denominators, simply add the numerators. The sum will become the numerator of your answer. The denominator will remain the same.

$$\frac{1}{3} + \frac{4}{3} = \frac{1+4}{3} = \frac{5}{3}$$

To subtract fractions, you must also find the common denominator. Then subtract the numerators to find the remainder. The denominator will remain the same.

$$\frac{7}{8} - \frac{5}{8} = \frac{7-5}{8} = \frac{2}{8}$$

To add and subtract mixed numerals, rewrite the numerals as improper fractions. Then follow the process outlined above.

$$3\frac{1}{4} - 2\frac{1}{2} = 3\frac{1}{4} - 2\frac{2}{4} = \frac{13}{4} - \frac{10}{4} = \frac{3}{4}$$

Adding and subtracting fractions is impossible without first writing the fractions with common denominators.

Multiplying and Dividing Fractions

Multiplying

To multiply a fraction by a whole number, change the whole number to a fraction by placing it over a denominator of one. (This does not change the value of the whole number.) Multiply the numerators then multiply the denominators to get the product.

$$\frac{1}{2} \times 1 = \frac{1}{2} \times \frac{1}{1} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$$

$$\frac{2}{7} \times 3 = \frac{2}{7} \times \frac{3}{1} = \frac{2 \times 3}{7 \times 1} = \frac{6}{7}$$

$$\frac{8}{9} \times 6 = \frac{8}{9} \times \frac{6}{1} = \frac{8 \times 6}{9 \times 1} = \frac{48}{9} = 5\frac{3}{9} = 5\frac{1}{3}$$

Change improper fractions to mixed numerals. Be sure the fraction part of the mixed numeral is written in the lowest possible terms.

To multiply one fraction by another fraction, multiply the numerators. Their product will become the new numerator. Next, multiply the denominators. Their product will become the new denominator.

multiply the numerators

$$\frac{1}{2} \times \frac{1}{3} = \frac{\quad}{\quad} = \frac{1}{6}$$

multiply the denominators

$$\frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$$

$$\frac{4}{3} \times \frac{1}{11} = \frac{4 \times 1}{3 \times 11} = \frac{4}{33}$$

To multiply mixed numerals (see p. 40) by fractions, change the mixed numerals to improper fractions. Then multiply the fractions.

change the mixed numeral to an improper fraction

$$\bullet \times \frac{2}{3} = \bullet \times \frac{2}{3} = \frac{13 \times 2}{7 \times 3} = \frac{26}{21} = 1\frac{5}{21}$$

$$\bullet \times 3\frac{1}{2} = \bullet \times \frac{7}{2} = \frac{17 \times 7}{8 \times 2} = \frac{119}{16} = 7\frac{7}{16}$$

Dividing

To divide a fraction by a whole number, change the whole number to an improper fraction with a denominator of one. Invert the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div 2 = \frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \qquad \frac{2}{7} \div 3 = \frac{2}{7} \div \frac{3}{1} = \frac{2}{7} \times \frac{1}{3} = \frac{2}{21}$$

To divide a whole number by a fraction or to divide a fraction by another fraction, ***invert*** the divisor fraction. Then multiply the fractions.

$$\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \bullet = \frac{1 \times 3}{2 \times 1} = \frac{3}{2} = 1\frac{1}{2} \qquad 7 \div \frac{6}{8} = \frac{7}{1} \times \frac{8}{6} = \frac{7 \times 8}{1 \times 6} = \frac{56}{6} = 9\frac{2}{6} = 9\frac{1}{3}$$

Invert the divisor fraction and multiply

To divide a mixed numeral by another mixed numeral, first change the mixed numerals to improper fractions. Then invert the divisor fraction and multiply.

$$\frac{1}{2} \div 2\frac{1}{3} = \frac{9}{2} \div \frac{7}{3} = \frac{9}{2} \times \frac{3}{7} = \frac{27}{14} = 1\frac{13}{14} \qquad 7$$

$$7\frac{6}{8} \div 6\frac{1}{3} = \frac{62}{8} \div \frac{19}{3} = \frac{62}{8} \times \frac{3}{19} = \frac{186}{152} = 1\frac{34}{152}$$

Turn it Upside Down: Inverting

Inverting a fraction means turning it upside down, or reversing the numerator and the denominator.

$\frac{1}{3}$ inverted is $\frac{3}{1}$ $\quad$ $\frac{6}{8}$ inverted is $\frac{8}{6}$

Inverting a whole number means to make it the denominator of a fraction with 1 as the numerator. 3 inverted is ⅓, 7 inverted is ⅐.
So, to solve the problem ⅓ ÷ 3,

invert 3 or $\frac{3}{1}$ to $\frac{1}{3}$

then $\frac{1}{3} \times \frac{1}{3} = \frac{1 \times 1}{3 \times 3} = \frac{1}{9}$